Mathematics Lessons for Grades 9-12

"Go with the Flow: Using Derivatives to Describe Storm Water Runoff Rates"

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Discipline: AP Calculus
Grade: 11 to 12

NCTM Standards:
1) Solve problems that arise in mathematics and in other contexts
2) Apply and adapt a variety of appropriate strategies to solve problems.
3) Use the language of mathematics to express mathematical ideas precisely.
4)College Board AP Calculus:
Understand the meaning of the derivative in terms of a rate of change and local linear approximation and they should be able to use derivatives to solve a variety of problems.

Purpose/Goal
1. Explain how engineers use derivatives to solve engineering problems.
2. Generate a graph representing storm water runoff data to create a graph representing the derivative of that function.
3. Compare instantaneous rates of change in runoff across three different graphs, representing different watershed scenarios and relate these different values to watershed characteristics.
4. Recommend appropriate storm water management technologies for reduction of urban runoff in a city such as Cincinnati, and how these management practices might impact the hydrographs they have created.

Context
No prerequisite knowledge is necessary for this lesson. Basic information on water runoff and watersheds can be found on USGS and USEPA websites. Students have trouble understanding the difference between discrete data (data point they collected) and a continuous function. Stress that we can only take derivatives of continuous functions. This is why we generate a best-fit curve, generating a continuous function.

Website
For worksheets and a description of how to make watershed models (urban, sub-urban, and rural) please visit:
http://www.eng.uc.edu/step/ginalamednellalessons.html
http://www.eng.uc.edu/step/ginalamednellalessons.html
http://apcentral.collegeboard.com/
apc/public/repository/ap08_calculus_coursedesc.pdf
http://www.sd1.org/education/fieldTrips.asp#f4

Motivation
A good introductory discussion could involve tracing where water travels when it hits the earth's surface and how we might describe surface water runoff rates in mathematical terms. Additionally, this is a great lesson to discuss how older cities use combined sewer overflows and how reducing water runoff in urban areas is necessary to protect the quality of our surface waters.

Description
The purpose of this unit is for students to discover how engineers use derivatives to solve real-world engineering problems (Careers). Students will utilize urban, sub-urban, and rural storm water runoff data, to generate three different mathematical functions. Using the derivative students will discover differences in instantaneous rates of change of water runoff within these different settings (Application to the real-world). Students should be able to correlate water runoff rate data to watershed characteristics, identify challenges associated with increased runoff rates in an urban setting such as Cincinnati (Societal Impacts), and suggest appropriate best management practices for its control. Finally, this unit will conclude with a trip to the national,

award-wining Sanitation District 1, for a tour of storm water management technologies utilized for reduction of urban runoff. At the end of this unit, students should be able to feel comfortable using derivatives to model change within environmental systems.

Science: Students will be exposed to hypothesis driven research, data collection, and data analysis.
Technology: Students will be using Microsoft Excel to generate graphical representations of their collected storm water data and to generated the slope of the tangent line.

Engineering: Students will learn how environmental and civil engineers design best management practices to reduce stormwater runoff. Students will also learn how engineers use derivatives to answer questions about environmental phenomena.
Mathematics: Students will apply the derivative to describe differences in storm water runoff rates in urban, sub-urban, and rural watersheds.

Assessment
Please see the written pre/post assessment and worksheets at http://www.eng.uc.edu/step/ginalamednellalessons.html. Additionally, some essential questions to ask during formative assessments are:
Essential Questions:
1. Name at least two differences in the derivative of the storm water runoff function for each of the three watershed scenarios.
2. What watershed characteristics result in the differences rate of change of storm water runoff within each of these different watershed scenarios?
3. Explain how you might use derivatives to solve other watershed related problems?

Follow-Up Activities
Further exploration could include using the models to measure runoff and calculate the volume of a runoff event using integration.