ACTIVITIES IN ALGEBRA:
IS GK-12 EFFECTIVE FOR HIGH SCHOOL ALGEBRA STUDENTS?

Andrea Knapp
Illinois State University GK-12 PRISM Project
akknapp@ilstu.edu

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Instructional activities and student handouts are available at www.gk12.ilstu.edu under Andrea Knapp's 04-05 lessons. See "Factoring Unit".

ABSTRACT

This study assessed the effectiveness of one Graduate Teaching Fellows in K-12 Education (GK-12) activity-based unit of instruction in a high school algebra class. Empirical evidence supporting the success of the GK-12 program for students is sketchy at best, and this study assists in closing this gap in research. Success of the GK-12 program was addressed through the study in terms of student understanding, enjoyment, and awareness of the usefulness of mathematics. Both quantitative and qualitative measures were employed in comparing an experimental class of 20 students with a control class of 16 students. Five focus students from each class were also interviewed for comparison purposes. Both classes consisted of 10th-12th grade students in their second year of a two-year algebra sequence at a suburban Midwest high school.

Success of the GK-12 program was documented in several domains. First, the experimental class's enjoyment and awareness of the uses of mathematics improved significantly, particularly demonstrated through willingness to participate. In addition, the experimental class was found via the posttest and qualitative measures to grow in conceptual understanding of mathematics. Although the experimental students' procedural understanding dipped, overall quiz score grades remained relatively unchanged. In keeping with the results of the study, expansion of and wide participation in the GK-12 program is recommended. Further study is needed to assess multiple GK-12 lessons for longer periods of time.

I. Introduction

Dr. Hans Freudenthal (1971) posed the question, “What is mathematics? What is it to the human mind?” as he wrote the article, Geometry Between the Devil and the Deep Sea”. It would do mathematics teachers well to take a moment to swim up from the deep sea of grading papers, parent visits, and lesson plans to consider this question. “What is mathematics?” writes Freudenthal, “Of course you know that mathematics is an activity. It is an activity of solving problems, of looking for problems” (p. 413).

Unfortunately, most mathematics classrooms today are a far cry from mathematics as a relevant activity. In direct contrast to NCTM’s recommendations, conventional classrooms rule the day, leaning heavily on memorization of facts, computational skill development, and rote application of formulas (Raghavan, Sartoris, & Glaser, 1998; Silver & Lane 1995). To show for our teaching practices, we find America’s middle and high school students falling well behind students from other industrialized nations in mathematics and science, including nations such as Japan, Canada, France, Australia, Hungary, and Ireland (TIMSS, 1996). Stigler and Heibert (1999) noted that on the Third International Mathematics and Science Study, “…in eighth grade mathematics, twenty of the 41 nations scored significantly higher, on average, than the United States” (p. 6). High school students fared worse, scoring below average in advanced mathematics and math/science literacy (Mullins et al., 1998).

To address this problem, the National Science Foundation has begun a program to improve the instruction of mathematics and science in the U.S. Nationwide, the GK-12 program awards grants to universities to place graduate students from disciplines of science, technology, engineering, and mathematics (STEM) into K-12 classrooms for the purpose of contributing to the national effort to address challenging issues in K-12 education. GK-12 Fellow activities include jointly designing and delivering K-12 science and mathematics instruction with classroom teachers. Fellows provide detailed content knowledge in applying scientific reasoning to real-world problems. Furthermore, it integrates research and teaching, as well as enhances teaching and curriculum selection/development skills for all participants through collaboration” (Moore, 2004). GK-12 is a relatively new program, beginning its first project in 1999. Currently, GK-12 projects are located at 118 sites in 41 states. Expected outcomes of GK-12 are as follows:

- opportunities for K-12 students to increase STEM content knowledge and skills and to work with STEM professional role models with whom they can relate
- improved relevance of science and mathematics content taught in K-12 schools
- graduate students serve as resources for K-12 teachers (Lundmark, 2004)
- strong partnerships between institutions of higher education and local school districts
- opportunities for classroom teachers to become more knowledgeable about and confident with STEM content and concepts
- professional development opportunities for teachers
- documentation of project outcomes to provide a research base to inform development of other GK-12 like activities and partnerships

II. Rationale

My experience as a high school teacher allowed me to see the benefits of GK-12, yet too few teachers seemed compelled to embrace this type of teaching. In performing this study, I wanted to provide empirical evidence that highlighted the benefits of teaching through realistic activities, and benefits of GK-12 in particular. I learned of the need for evaluation of GK-12 programs when I attended the national GK-12 conference in March of 2004.

Although some evaluation of GK-12 has been done, much of it has focused on the program's impact on teachers and Fellows. I found research on the impact on students to be sparse. Several examples of GK-12 success that I did find include the following:

- Participating middle schools in Detroit had a mean score 20% higher in mathematics than non-GK-12 schools on the Detroit Public Schools' Essential Skills Attainment Test ("NSF GK-12 Nuggets").
- An inner city school in Seattle saw 4th grade students rise from 1% meeting state standards in the first year of GK-12 involvement to 45.5% meeting in the second year, and 60% meeting standards in the third year (Woodin, 2004).
- The GK-12 program at the Universidad Metroploitana in Puerto Rico reported an 80% increase in the number of pupils passing a GK-12 influenced science course (NSF reports).
- Finally, in Illinois, Gabric et al. (n. d.) found evidence of increased complex thinking and problem solving skills, improved literacy, and increased awareness and interest in science careers.

In reviewing NSF reports for empirical evidence, I found that much of the data has been collected about science and about middle school students ("NSF Reports", n.d.). Thus, I felt that a need existed to gather evidence for the success of GK-12 on high school mathematics students.

III. Theoretical Perspective

I chose Social Constructivism as the theoretical perspective for my study because it reflected what I saw happening through GK-12 interaction in classrooms. In choosing social constructivism, I leaned heavily on the work of Dutch mathematicians such as Hans Freudenthal who founded Realistic Mathematics Education (RME) in the Netherlands. RME emphasizes that teachers should introduce mathematics based on students' real and familiar experiences. Students then, with the teachers help, reinvent real-world phenomena mathematically to create increasingly complex mathematical models to investigate. As I mentioned earlier, Freudenthal held that mathematics is a human activity. You don't learn to cycle, swim, or ski from a text, and neither do you learn mathematics that way (Freudenthal, 1971).His research in geometry education revealed that students are naturally inquisitive, and thus he believed that students' inquiry should be the point of entry for mathematics. This idea is consistent with the GK-12 model.

In addition to Freudenthal, I also looked to Jean Piaget and Lev Vygotsky to support my framework. Piaget concluded that all knowledge is constructed through innate cognitive structures (Noddings, 1990). Furthermore, Piaget insisted that certain logical structures develop through a coordination of *actions* [emphasis added] (Piaget, 1971).

Vygotsky, on the other hand, held that language and interpersonal or social interactions underpin intellectual development. He saw both language and mathematics as tools through which children develop a superior form of activity or learning. I saw social constructivism as coupling the ideas of Piaget and Vygotsky. Terwel (1999) noted that social constructivism embodies both construction of knowledge and social interaction with it, 1999). She listed the following attributes of social constructivism: active and strategic knowledge acquisition; appreciation for diversity of expertise, learning styles, and interests; and collaboration and reflection in a community of inquiry. Using social constructivism to guide my theoretical framework, I arrived at the following research questions.

1. Do students who participate in GK-12 activity-based lessons demonstrate greater mathematical understanding than those in a traditionally taught algebra classroom?
    - How do student scores on quizzes, tests, and exams compare across classrooms? Is there a statistically significant difference?
    - What opportunities did students in both classes have for critical thinking and problem solving? How is this reflected in interview responses and classroom observations?

2. Do students taught with GK-12 activity-based lessons demonstrate greater enjoyment of mathematics?
    - Are they more willing to participate and persist in mathematical activities than students in the traditional class?
    - In what ways do students express enjoyment in survey responses, interviews, and classroom observations?

3. Do students taught with GK-12 activity-based lessons demonstrate a greater awareness of mathematics and its usefulness in their lives?

IV. Methodology

For the methodology of this comparative study, I employed both quantitative and qualitative measures. The participants included 36 10-12th grade students enrolled in the second year of and Algebra 1 sequence at a large mid-west high school. The experimental class contained 20 students, and the control class contained 16. Five students from each class were chosen as focus students for initial and final interviews. Focus students represented a variety of achievement levels.

The procedure for this study included the teaching of 15 GK-12 lessons on factoring trinomials over the course of a month. I was the PRISM fellow that collaborated with the teacher to provide the lessons. The experimental class was videotaped 5 times, and I took reflective notes after each lesson. The traditional class was observed seven times and videotaped twice. Sources of data included a pre and post test, observations, initial and final interviews, initial and final attitude surveys, and scores from five quizzes.

The control class followed the same daily pattern of instruction. Warm Up was followed by Notes, and Homework time allotted was 30 minutes. Quizzes were given every Friday. In contrast, the experimental class generally began with a discussion about real-world applications of the topic to generate interest. A 30-minute activity followed with students working in pairs. The lessons concluded with reflective discussion, making up problems, presenting outcomes, and practicing problems. Students were probed with the how and why of their answers.

V. Results (E= Experimental class; C = Control class; Q = Quiz)

A. Baseline – to determine initial levels of student achievement to compare classes.

1. Pretest: E scored 64%, C scored 67 %
2. Quiz 1: E scored significantly lower ($p = .03$)

Therefore, by measures of the pretest and Q1, E students began with slightly lower achievement levels.

B. Procedural Understanding – ability to correctly factor with relative ease and speed.

1. Quiz Scores – weekly, same for both classes. If students did not complete their homework, they were not allowed to take the quiz.
    a. Figure 3. I compared classes using a 2-sample t test. The study began Tuesday of the week before Q2, ended the Tues of the week before Q5.
    b. E significantly lower on Q1, Q2, Q3, not on Q4, Q5
    c. Significant rise in E scores from Q3 to Q4, not C
    d. Significant drop in E scores from Q4 to Q5, not C
    e. Explanation of Q's: Q1, Q2 were review; Q3 was the 1st quiz over factoring trinomials; Q4 students becoming adept at factoring trinomials and had the most factoring problems; Q5 was a mixture of all types of factoring. All Q's had some review problems.
    f. Conclusion – Participating in GK-12 lessons did not adversely affect grades on Procedural Quizzes – perhaps continued GK-12 interaction would have reduced the achievement gap further.
2. Quiz Ranges - scores of 0 dropped since students who did not complete their homework were not allowed to take the quiz.
    a. Figure 4. On 1st QZ, 31 point gap in ranges, last QZ, only 4 pts
    b. Also, E had range of 85 on Q1 and 54 on last
    c. C had range of 54 on Q1 and 50 on last
    d. Thus, activity-based GK-12 lessons decreased the spread in scores and narrowed the achievement gap of the E class. Based on the ranges, E became stronger mathematically.

4. Posttest – Procedural factoring problems and conceptual questions.
    a. Example question: When would 2 factors have opposite signs such as (?+?)(? -?)? Why are opposite signs necessary?
    b. E 64%, C 71%, E not significantly lower

C. Conceptual Understanding – knowing how to factor, why it worked, for what purpose. Opportunities for problem solving and critical thinking--

1. Post test – E class significantly lower on procedural questions ($p = .01$), significantly higher on conceptual problems ($p = .03$); overall, no significant difference. Losses in procedural offset by conceptual.

2. Baseline for Critical thinking, Problem solving–knowing when and why to apply procedures/knowledge, making sense of complicated situations, generating hypotheses, critically examining events and selecting appropriate strategies for solving problems (Silver & Lane, 1995). Prior to the study, focus students were asked questions such as:

a. What do you do if you get stuck on a math problem? How do you feel?

i. 6/10 reported negative feelings.

ii. Most popular strategy – Ask the teacher vs. reasoning through or discussing with a peer.

b. T/F You must follow steps to get correct answers in math.

i. All but 1 said True.

c. Describe how you factored $2x^2 – 4xy^2 + 6x^2y$ on the pretest.

i. Procedural explanations. Lack of understanding of procedures in two student responses

- "Pull it apart…Just take it step by step"…(Lea)
- "I found what all they had in common and then put it down there and then I put what was left in the parentheses. It's hard for me to describe how I work it because I can do it, but I can't really explain it. It's just something I do." (Homer)

When Lea had steps to follow, she could factor, otherwise she often became confused

This corroborates Ball's conclusion that once a rule is forgotten it's not easily retrievable without concepts to support it (Ball, 1990).

d. Conclusion – Prior to intervention, students were given few opportunities to think critically or problem solve.

3. Critical Thinking and Problem Solving after intervention.

a. E class given opportunities to problem solve in real-world contexts. – Kathleen and football problem $0 = 16t^2 – 90t + 11$. Kathleen emerged from her dependence on the teacher. She asked for help and I rephrased the problem. She initially gave up, but then persevered with input from her peers. She won a contest to invent and factor her own trinomials.

b. Final interview - "What helped you understand today's lesson most?"

Lea, from the C class, looked back in her notes to decide what signs to use. Procedural notes were given that day on when the last number is positive. The teacher told them the signs to use. A student asked, are they all like this? Her answer for that day was "Yes". On the other hand, the E class was encouraged to think critically about signs using algebra tiles. Often they were questioned, "Why?" about signs they chose.

c. Final interview – "Show/Tell me as much as you can about $x^2 - 5x + 4$" The E focus students all got it. Lea, who was identified by the teacher as having high achievement, kept trying to apply the rule for pulling out a greatest common factor (GCF). C students associated the direction "factor" to mean pull out the GCF, leading to the question of whether they knew what it meant to factor a polynomial.

d. "What does it mean to factor?" – initial and final interviews. Initial interview– most students said to break apart or find what factors in common. One student said, "I don't know. Once again, I just do it. I don't know."

Final interview– E class – 1 student said, "to simplify the equation to multiplication form. Another student also revealed that she realized that factoring rearranged the polynomial to a different form.

-C class had little conceptual understanding of what they'd been doing the last month. Homer, again said, "I don't know. Dora said, "To figure out the problem." The other three referred to pulling out a GCF.

e. "How would you describe factoring to a classmate who missed the last unit? Lea (C class) referred to taking it step by step. Homer (C class) - "[You] don't have to really do much thinking."

f. Cognitive Activity Levels. – cross coded with a graduate student
C – consistently 0-2
E – reached levels of 3 or 4 during 82% of the lessons that I coded
E class reached 3-4 in real-world lessons when students created problems, constructed knowledge, and presented solutions. For example, they tossed beech balls above motion detectors to examine ball's distance vs. time and the resulting equations and graphs. One student asked why factors would by (-)(-) when roots had to be positive. He was thinking critically about meaning of solutions resulting from factoring. Students also reached higher levels of cognitive activity as they constructed factoring rules from algebra tiles.

Thus, the E class had more opportunities to critically think and problem solve. The levels of Cognitive Activity provided evidence that the GK-12 lessons resulted in greater mathematical understanding.

In contrast, the C class was told to copy notes exactly. Thus, based on my observations, it is easy to see why the E class did better on conceptual understanding and the C class on procedural. The E class had a much better grasp of what they were doing when they factored and why. Consequently, they experienced growth in critical thinking, problem solving, and overall mathematical understanding of factoring.

D. Enjoyment

1. Pre/post attitude survey (over factoring unit) – rated statements 1-5 such as "I have usually enjoyed studying mathematics in school." and "Math is dull and boring" – reverse coded. The survey was adapted from Tapia (1996).

a. Prior to study, E significantly lower than C ($p = .07$), after intervention, there was no longer a significant difference.

b. E score rose significantly ($p = .10$), C Class did not rise significantly.

2. Qualitative observations – students smiling laughing, conversing as worked with algebra tiles and CBR's.

3. Interview Responses – "How did you like factoring compared to other algebraic topics, Why?"

i. E students gave more positive explanations about factoring than C students. In fact, Lea said factoring was kind of boring. Wen said factoring was more complicated than other topics.

It is interesting to compare these responses with the initial interview in which students reported liking math when it was easy, they saw its usefulness, and when they liked the teacher. Based on the significant increase in enjoyment of the E class, I speculate that enjoyment is more tied to meaningful, realistic activity-based instruction than it is to easy content.

E. Willingness to participate.

- E students readily participated in class discussions about the applications of Quadratic functions. (Ie – free falling , shooting a stunt man from a canon, throwing a football. On the day in which I presented a free fall situation, I discussed Lt. I. M. Chisov's survival of a free fall from an airplane (*Chuteless Jumps*, 2003). He plowed 3 feet into the snow. Chuck said he didn't know anything about free falling, but that he wanted to do it. One student ventured a guess that Lt. Chisov's fall would have taken 3.5 second. Chuck thought a little over a minute. The equation was $h = -16t^2 + 21980$ with $t = 37$ seconds (*Chapter 5*, n.d.). The students seemed to appreciate knowing that a purpose for factoring is to find roots, which I showed them graphically on the board and the next day on the graphing calculator. Real world contexts lent themselves to discussion. General interest and willingness to participate followed.
- Students seemed to enjoy creating their own problems. For example, in the football lesson, April, a normally quiet and uninvolved student,

invented 8 factorizations. Max passed her up with 9. One day I challenged them to come up with the hardest problem they could think of for the class to factor. Jude, a special ed student immediately accepted my challenge by stating $121x^2 - 100$, and Max immediately factored it. The next day, to low achieving students invented problems for the lass to factor. Students showed that they liked having races to factor in their heads by flailing their hands and asking for a prize.

- More students worked to the end without prompting.
    i. C class – students became more restless and disengaged by the end of class. The teacher had to prompt them to work, wake up. She often reminded them, “It’s not 5 minutes yet”, and threatened detentions for students who completed only one or two homework problems in the 30 minutes allotted.
- In response to the long homework assignments, I heard students say, “Do we have to do all of them? Oh…!” They shook their wrists, accusing the teacher of causing them Carpel Tunnel Syndrome. On one occasion, a student was heard to say, “I’m on [problem number] 25. My arm’s killing me, man.” Clearly, many control students did not enjoy having to work 25-30 procedural problems every day. Hence, observations supported the survey and interview responses of this study revealing that experimental students definitely benefited from GK-12 lessons in terms of enjoyment of mathematics and willingness to participate in daily mathematics activities in the classroom.

F. Math Awareness (MA)

1. Survey: 8 questions about MA. Ie. “A strong math background could help me in my professional life.” Survey items were adapted from Tapia (1996).

    a. MA score grew significantly for E class. ($p = .09$)

    b. Write as many examples of where math is used in real life as you can.”

        i. Initial survey – E significantly lower, Final – no significant difference.

    c. Final interview – Give as many real world applications of FACTORING as you can.

        i. E gave significantly more examples. $p = .09$ (3 of the control focus students gave 0)

    d. Conclusion - In conclusion, students in the experimental class benefited from a realistic, activity-based, problem solving curriculum in several ways. First, students grew in mathematical understanding, particularly in conceptual understanding. Students in the experimental class had many more opportunities to think critically and problems solve, resulting in higher levels of cognitive activity during class time. Furthermore, experimental students grew in enjoyment of mathematics and demonstrated more willingness to participate in mathematics class, especially when compared to the control class which did not appear to enjoy the factoring unit. Finally, the experimental class grew in awareness of the usefulness of mathematics while participating in GK-12 activity-based lessons.

VI. Limitations

Why did procedural understanding of the E class suffer while overall Quiz grades rose, relative to the control class? I submit that increased enjoyment and participation resulted in better grades, especially in light of increased participation in the last 5 minutes of class. Hopefully procedural scores would increase over time.

A. Time – only 1 month for study. It took time to adjust to new method of instruction and an additional teacher. It also took time for researcher to become acclimated to culture of class and make appropriate assessment decisions.

B. Assessment favored the C class – The quizzes were highly procedural, and the E class had trouble finishing them. If the quizzes had been shorter and more conceptual, E scores may have been higher.

C. Limited Scope – this study involved only one GK-12 program, 1 fellow, and 1 teacher

D. Further study needed on a larger scale and for longer periods of time.

E. Further study could investigate the effects of GK-12 on a teacher and her students when a critical mass of teachers in a school became involved with GK-12. I could ask, Does the teacher continue to use activity-based units when not helped by a GK-12 fellow?

VII. Implications

A. School districts – encourage wide participation of math and science teachers

B. GK-12 universities expand and sustain programs, perhaps as part of teacher education programs or as elective options for graduate students.

C. NSF should expand GK-12 programs to more universities so students nationwide may benefit.

If students in one classroom could make gains in understanding, enjoyment, and awareness of mathematics in just one month of interaction with GK-12 activity-based lessons, consider the mathematical advances that could be made in a whole school, a community or a nation. The GK-12 partnership program provides a promising model for improving mathematics instruction. Hopefully this study has assisted in highlighting the success of Graduate Teaching Fellows in K-12 Education.

For more information, see

Knapp, A. K. (2005). *Activities in algebra: Is GK-12 effective for high school algebra students?* Unpublished master's thesis, Illinois State University, Normal.

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